How to Explain Empirical Efficiency of Non-Associative Aggregation Operations?

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Need to describe experts' uncertainty in computer-based systems. A significant part of our knowledge comes from experts. Experts are often not absolutely confident about their statements. For example, a medical doctor can say that certain of an X-ray images indicate possible cancer. However, the doctor may add that there are cases when other diseases show similar pattern. So, additional tests are needed before we start a treatment. To adequately represent the expert's knowledge in a computer-based system, we need to take into account the expert's uncertainty.

To computers, a natural language is the language of numbers, this is what they were deigned for. So, a natural idea is to describe the expert's degree of certainty in a statement by a number. In a computer, 0 usually means "false" and 1 usually means "true". Thus, it makes sense to describe intermediate degrees of certainty by numbers between 0 and 1.

Need for aggregation operations. When a computer-based system makes a recommendation or a conclusion, it usually uses several statements $S_1, \ldots, S_n$. Our degree of confidence in this recommendation is equal to our confidence that all these statements are true: $S_1 \,\&\, \ldots \,\&\, S_n$.

In some cases, several different statements S_i lead to the same conclusion. In this case, we are interested in our degree of confidence that one of them is true, i.e., that $S_1 \vee \ldots \vee S_n$.

There are exponentially many different combinations of statements: 2^N, when we have N statements. It is not feasible to ask the expert about all possible combinations. So, we need to estimate our degree of confidence in $A \,\&\, B$ and $A \vee B$ based on the degrees of confidence a and b in A and B. The corresponding algorithms $f_\&(a,b)$ and $f_\vee(a,b)$ are known as "and"- and "or"-operations.

Most systems use associative operations, but sometimes non-associative ones work better. In general, the statement $(A \,\&\, B) \,\&\, C$ and $A \,\&\, (B \,\&\, C)$ mean the same. So, it is reasonable to select $f_\&(a,b)$ for which the estimates degree of confidence in these statement are equal: $f_\&(f_\&(a,b),c) = f_\&(a, f_\&(b,c))$. In mathematics, such operations are known as associative. There are many such associative operations: $\min(a,b)$, $a \cdot b$, $\max(a+b-1,0)$, etc.

A recent study showed that in some risk-related applications, we get better results with non-associative operations

$$\frac{\max(a+b-1,0)+\min(a,b)}{2} \text{ and } \frac{a \cdot b+\min(a,b)}{2}.$$

In this talk, we explain this empirical success.