Why Ratio Bias?

Timothy J. Gonzales, Ricardo Medina, Sajid Rashid, and
Vladik Kreinovich
Department of Computer Science, University of Texas at El Paso
500 W. University, El Paso, TX 79968, USA
tjgonzales@miners.utep.edu, remedina5@miners.utep.edu
srashid2@miners.utep.edu, vladik@utep.edu

Formulation of the problem. In general, people prefer alternative with higher probability of success – and lower probability of failure. However, there are exceptions to this reasonable idea. Namely, when the same probability comes from a larger sample, people prefer this alternative. For example, suppose that we have two alternatives: one of them was successful 90 times out of 100, while the second one was successful 9 times out of 10.

The probability of success is the same in both cases, but most people prefer the second alternative. This is especially true in health-related decisions. This phenomenon is known as the *ratio bias*. It is an important case of more general *numerator bias*, when people pay more attention to the numerator than to the ratio. How can we explain this bias?

It is not probabilities that are equal. We said that the probability is the same in both above cases. However, this is not exactly correct. What is the same is the *frequency*, and probability is, in general, somewhat different from the frequency.

For example, a fair coin lands heads with probability 0.5. This means that, on average, it falls heads in half of the cases. However, it does not mean that if we flip the coin 100 times, it will fall heads exactly 50 times. This is even more clear if we flip a coin 101 times: here, exactly half would mean 50.5 times – and 50.5 is not even an integer.

If we have an event with probability p, then its average frequency is indeed p. However, the frequency may deviate from probability.

How to describe the deviation of frequency from probability? In statistics, we usually gauge such deviation by its root mean square value σ known as *standard deviation*: $\sigma = \sqrt{E[(f-p)^2]}$. It is known that for repeating events with probability p: $\sigma(p) = \sqrt{(p\cdot(1-p))/n}$. In principle, deviation can be as large as possible, but the probability of deviations much larger than a few signa's is very small. However, in statistics, when we make decisions, we usually ignore such rare deviations. Thus, we assume that deviations do not exceed $k\cdot\sigma$ for some small k, usually $k=2,3,6$. For $k=2$, the probability of exceeding this bound is 5%, for $k=3$, it is 0.1%, and for $k=6$, it is 10^{-6}%. So, based on the observations, we do not get the exact value of the probability.

What we conclude is that the actual (unknown) probability is somewhere in the interval $[p-k\cdot\sigma, p+k\cdot\sigma]$. Since we do not know the exact probability, we do not know the exact value of the expected gain $p\cdot g_0$. All we know is that this gain is somewhere in the interval $[\underline{u},\overline{u}] = [(p-k\cdot\sigma)\cdot g_0, (p+k\cdot\sigma)\cdot g_0]$.

How to make a decision under such interval uncertainty? When we know the exact gain of different alternatives, we select the alternative with the largest expected gain. How can we make a decision if we only the interval of possible gains? In such cases, decision theory recommends to use so-called Hurwicz criterion. Namely, we pick some value α between 0 and 1, and select the alternative for which the following combination is the largest: $u = \alpha\cdot\overline{u} + (1-\alpha)\cdot\underline{u}$. The coefficient α is known as optimism-pessimism coefficient.

We show that this idea can explain the ratio bias.