Maximal d-Elements of M-frames

Papiya Bhattacharjee

Florida Atlantic University

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Outline

1. Preliminaries
2. Primes and Minimal Primes
3. $Max(dL)$
4. The space $Max(dL)$
5. Ultrafilters of $\mathfrak{K}L^{\perp}$

Definition

1. A *frame* L is a complete lattice that satisfies a strong distributive law where finite infimum distributes over arbitrary supremum.
2. An element $c \in L$ is *compact* if $c \leq \bigvee a_\alpha$ implies that $c \leq a_{\alpha_1} \vee \cdots \vee a_{\alpha_n}$. The collection of all compact element of L is denoted by $\mathfrak{K}(L)$.
3. A frame is *algebraic* if every element in the frame is the supremum of compact elements.
4. A frame is said to satisfy the finite intersection property (FIP) if $c, d \in \mathfrak{K}(L)$ implies that $c \wedge d \in \mathfrak{K}(L)$.
5. For each $x \in L$, $x^{\perp} = \bigvee\{y \in L : y \wedge x = 0\}$.
6. $x \in L$ is dense if $x^{\perp} = 0$. A compact, dense element of L is called a unit.

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- An element $p \in L$ is *prime* if $p < 1$ and for all $a, b \in L$, $a \wedge b \leq p$ implies that $a \leq p$ or $b \leq p$.

- A prime element p is *minimal* if there are no other prime elements $q < p$. We denote the collection of minimal prime elements of L by $Min(L)$. Using Zorn's lemma we can show that primes and minimal primes exist in algebraic frames.

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- **Zariski Topology on** $Min(L)$**:**
Let $c \in L$.
Define $U(c) = \{p \in Min(L) : c \nleq p\}$

Lemma

Let $x, y \in L$. The following holds:

1. $U(x) \cup U(y) = U(x \vee y)$ *and* $U(x) \cap U(y) = U(x \wedge y)$.
2. $\bigcup U(x_\alpha) = U\left(\bigvee x_\alpha\right)$.
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The *Zariski topology* on Min(L) is the topology generated by the collection $\mathcal{B} = \{U(c) \mid c \in \mathfrak{K}(L)\}$.

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$Min(L)$ is a Hausdorff, zero-dimensional (base of clopen sets) space.

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Question: When is $Min(L)$ a compact space?

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Define the set-theoretic complement of $U(x)$, called $V(x) = \{p \in Min(L) : x \leq p\}$. The following holds for the operator $V(\cdot)$.

- $V(x) \cup V(y) = V(x \wedge y)$ and $V(x) \cap V(y) = V(x \vee y)$.
- $V(x) = Min(L) \Leftrightarrow x = 0$.

The collection $\{V(k) : k \in \mathfrak{K}(L)\}$ forms a basis for a topology on $Min(L)$, called the *Inverse topology*, and denoted by $Min(L)^{-1}$.

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Theorem

The following are equivalent for an M-frame L.

- *The Zariski topology on $Min(L)$ is compact.*
- $Min(L) = Min(L)^{-1}$.
- *For each $x \in \mathfrak{K}(L)$ there exists $y \in \mathfrak{K}(L)$ such that $x \wedge y = 0$ and $x \vee y$ is a unit.*
- *L is a complemented frame.*

Lemma on Ultrafilters

For an M-frame L, $Min(L)$ is in bijective correspondence with $Ult(\mathfrak{K}(L))$. In particular,

$$p \in Min(L) \text{ implies } F_p = \{c \in \mathfrak{K}(L) : c \nleq p\} \in Ult(\mathfrak{K}(L)),$$

and

$$U \in Ult(\mathfrak{K}(L)) \text{ implies } p(U) = \bigvee\{c^{\perp} : c \in U\} \in Min(L).$$

Additionally, $p(F_p) = p$ and $F_{p(U)} = U$.

Hence, the map $\Phi : Ult(\mathfrak{K}(L)) \to Min(L)$ defined by $\Phi(U) = p(U)$ is a well-defined bijection with $\Phi^{-1} = F_p$.

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Topologies on $Ult(\mathfrak{K}(L))$:

Given $c \in \mathfrak{K}L$, we define
$\mathfrak{U}(c) = \{U \in Ult(\mathfrak{K}L) : c \notin U\}$ and $\mathcal{V}(c) = \{U \in Ult(\mathfrak{K}L) : c \in U\}$.

- The *Wallman topology* on $Ult(\mathfrak{K}(L))$ is generated by the collection $\mathcal{B}_{\mathfrak{U}} = \{\mathfrak{U}(c) \mid c \in \mathfrak{K}(L)\}$.
- The *inverse topology* on $Ult(\mathfrak{K}(L))$, denoted $Ult(\mathfrak{K}(L))^{-1}$, is generated by the collection $\mathcal{B}_{\mathcal{V}} = \{\mathcal{V}(c) \mid c \in \mathfrak{K}(L)\}$.

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Let L be an M-frame with a unit. The following holds:

1. *The topological space $Ult(\mathfrak{K}L)$ is homeomorphic to $Min(L)^{-1}$.*
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1. An element $d \in L$ is a *d-element* if $d = \bigvee\{c^{\perp\perp} : c \in \mathfrak{K}(L), c \leq d\}$. The collection of all d-elements of L is denoted by dL.
2. $x \in dL$ if and only if given any $c \in \mathfrak{K}(L)$, $c \leq x \Leftrightarrow c^{\perp\perp} \leq x$.
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- dL is a frame, where meet is same as in L and join is given by $\bigvee^d S = d(\bigvee S) = (\bigvee S)_d$.

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Existence of $Max(dL)$:

- Suppose L is an M-frame that contains a unit u.

- Let $A = \{x \in dL : x < 1\}$. Obviously $0 \in A$, which means that $A \neq \emptyset$.

- Take a chain $\{x_\alpha\}$ in A and let $x = \vee x_\alpha$. Suppose $c \in \Re L$ and $c \leq x$. There exists a finite subcollection from the chain such that $c \leq x_1 \vee \cdots \vee x_n \leq x$. Hence $c \leq x_i \leq x$, for some i. Since $x_i \in dL$ it follows that $c^{\perp\perp} \leq x_i \leq x$. Therefore $x \in dL$.

- If possible let $x = 1$, then $u \leq x$. As before, $u \leq x_j \in A$ for some j. Since $x_j \in dL$, $1 = u^{\perp\perp} \leq x_j$, contradicting the fact that $x_j < 1$. Hence $x < 1$, proving that $x \in A$.

- By Zorn's Lemma, maximal d-elements exist in L.

- We use $Max(dL)$ to denote the (proper) maximal d-elements of L.

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Two important results:

Theorem

Let $x \in Max(dL)$, then x is maximal with respect to $u \nleq x$ for all units u of L. Conversely, if $y \in L$ is maximal with respect to $u \nleq x$ for all units u, then $y \in Max(dL)$.

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Two important results:

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Theorem

$Min(L) \subseteq dL$ and $Max(dL) \subseteq Spec(L)$.

We endow *Max*(*dL*) with the subspace topology from *Spec*(*L*), namely, the Zariski topology.

Define

$$U_d(x) = \{m \in Max(dL) : x \not\leq m\}.$$

The following hold for all $c, k \in \Re L$.

1. $U_d(c) = Max(dL)$ if and only if c is a unit.
2. $\bigcup U_d(c) = U_d(\bigvee c)$.
3. $U_d(c) \cap U_d(k) = U_d(c \wedge k)$.
4. $U_d(c) = U_d(c^{\perp\perp})$.

The collection $\{U_d(c) : c \in \Re L\}$ forms a basis of open sets for the Zariski topology on *Max*(*dL*).

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Theorem

Let L be an M-frame that possesses a unit u. The space $Max(dL)$ is a compact topological space.

Important Observation: In case of a **W**-object (G, u), $Max_d(G)$, w.r.t the Zariski topology is a compact, **Hausdorff** space. It so happens that to prove the Hausdorff condition, we require the property of "disjointification" which ℓ-groups possess. This is the first result in frames that seems to NOT be occurring parallel to that of ℓ-groups, since an algebraic frame does not satisfy disjointification always.

Question: If L is an M-frame with a unit, when is $Max(dL)$ Hausdorff?

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Question: If L is an M-frame with a unit, when is $Max(dL)$ Hausdorff?

- It is known that

$$Max_d(G) \cong Ult(Z^{\sharp}(G)), \text{ for } \mathbf{W}\text{-objects}$$

and

$$Max_d(X) \cong Ult(Z^{\sharp}(X)) \text{ for spaces}$$

where $Z^{\sharp}(X) = \{\text{cl int } Z : Z \in Z(X)\} \subseteq \mathcal{R}(X)$ (regular closed subsets of X)

- In frames, $\mathfrak{B}(L) = \{x^{\perp} : x \in L\}$ is analogous to $\mathcal{R}(X)$ for spaces.

Q: In frames, what is analogous to $Z^{\sharp}(X)$?

- Let $\Re L^{\perp} = \{c^{\perp} : c \in \Re L\} \subseteq \mathfrak{B}(L)$ be a subset.

Lemma

$\Re L^{\perp}$ is a sublattice of $\mathfrak{B}(L)$, with meet same as the meet of L and the join is given by $x^{\perp} \vee' y^{\perp} = (x \wedge y)^{\perp}$. Also, $\Re L^{\perp}$ is a bounded lattice

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Theorem

Suppose L possesses a unit u.

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2. *If $x \in dL$ is proper, then $\hat{F}_x = \{c^{\perp} : c \in \Re L, c \leq x\}$ is a filter of $\Re L^{\perp}$.*
3. *$\hat{F}_{x(G)} = G$, for every filter G of $\Re L^{\perp}$ and $x(\hat{F}_y) = y$, for every $y \in dL$.*

Let $\Phi_d : Ult(\Re L^{\perp}) \to Max(dL)$ be defined by $\Phi_d(U) = x(U)$,

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Next, we topologize $Ult(\Re L^{\perp})$ with the well-known Wallman topology: basic open sets are $M(I^{\perp}) = \{U \in Ult(\Re L^{\perp}) : I^{\perp} \notin U\}$, for $I \in \Re L$.

Finally, the main result:

Theorem

Let L be an M-frame with a unit u. The map $\Phi_d : Ult(\Re L^{\perp}) \to Max(dL)$ is a homeomorphism between the topological spaces $Ult(\Re L^{\perp})$, with respect to the Wallman topology, and $Max(dL)$, endowed with the hull-kernel topology.

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- If $U \in Ult(\Re L)$, then $U^* = \{c^{\perp} : c \in \Re L \setminus U\}$ is a prime filter of $\Re L^{\perp}$.

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Q: When can we have a well-defined bijection between $Ult(\Re L)$ and $Ult(\Re L^{\perp})$?

Answer: Complemented frames

Theorem

The following are equivalent for an M-frame L that possesses a unit u.

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We know,

$Min(L)^{-1}$ Hausdorff $\Leftrightarrow$ $Ult(\mathfrak{K}L)$ Hausdorff $\Leftrightarrow$ $\mathfrak{K}L$ normal lattice $\Leftrightarrow$ L Lamron

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Question: If L is an M-frame with a unit, when is $Max(dL)$ Hausdorff?

We know,

$Min(L)^{-1}$ Hausdorff $\Leftrightarrow$ $Ult(\mathfrak{K}L)$ Hausdorff $\Leftrightarrow$ $\mathfrak{K}L$ normal lattice $\Leftrightarrow$ L Lamron

Similarly,

$Max(dL)$ Hausdorff $\Leftrightarrow$ $Ult(\mathfrak{K}L^{\perp})$ Hausdorff $\Leftrightarrow$ $\mathfrak{K}L^{\perp}$ normal lattice $\Leftrightarrow$ L ??

THANK YOU