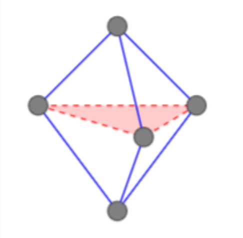
BLAST
9-13 June 2021
Las Cruces, NM
ONLINE

Lattices
Universal Algebra
Boolean Algebras
Set Theory
Topology



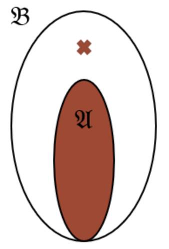
On The Networks of Large Embeddings

Tuğba Aslan   Mohamed Khaled   Gergely Székely

General Settings ...

Large embedding

$\mathfrak{A}$ is a large subalgebra of $\mathfrak{B}$.



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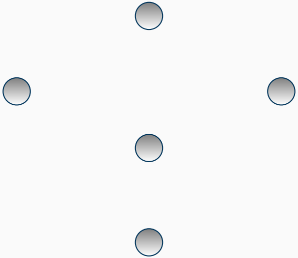

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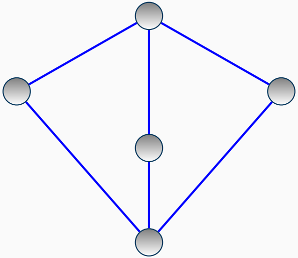

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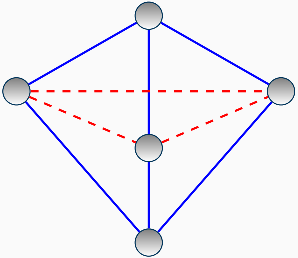
Dashed Red Edges: isomorphic algebras



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The class of all subgroups of A_4:

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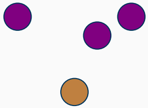

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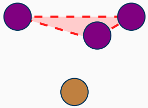

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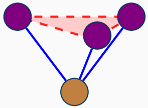

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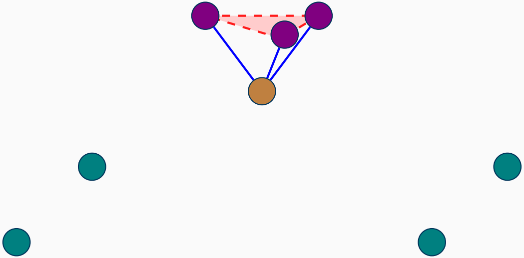

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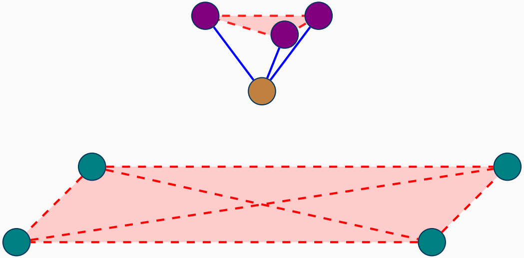

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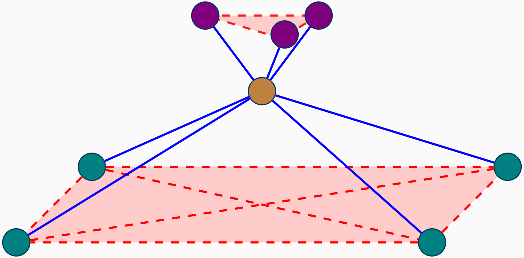

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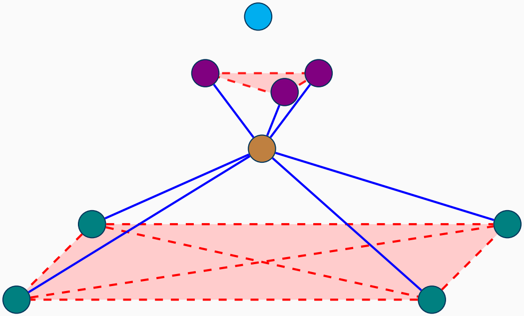

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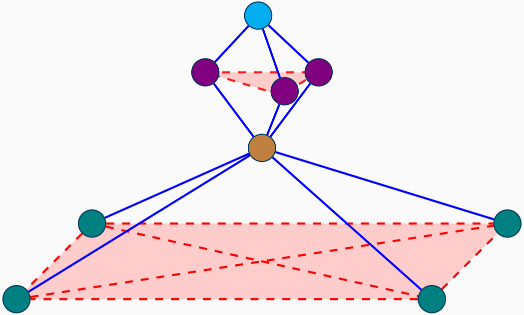

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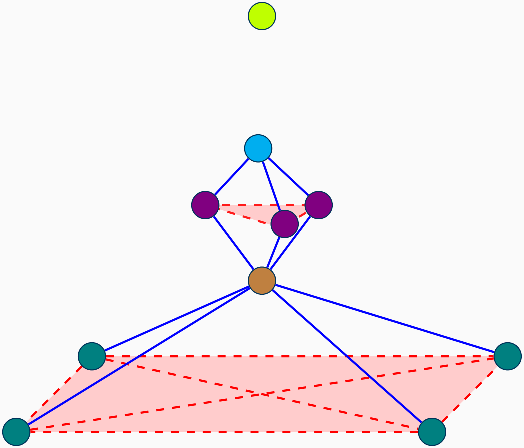

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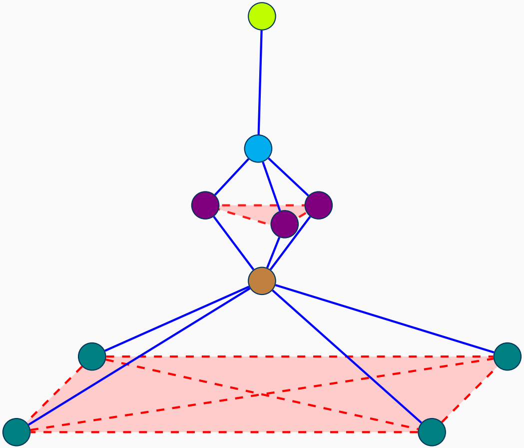

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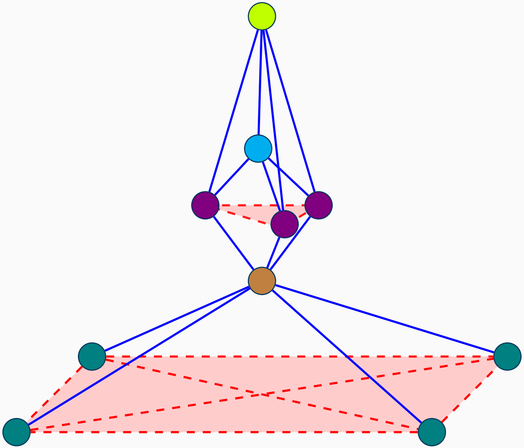

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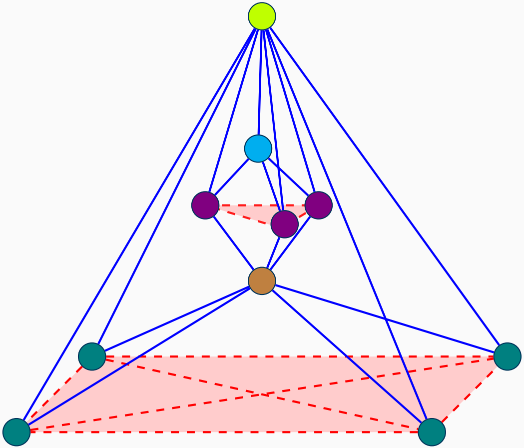

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- None of these groups is isomorphic to or embeddable in the other one.
- Both V_4 and Q_8 are largely embeddable into the central product of D_8 and Z_4.

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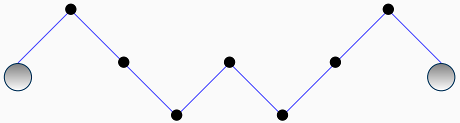

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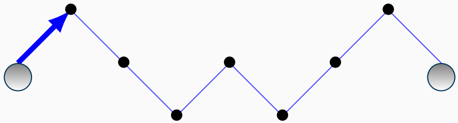

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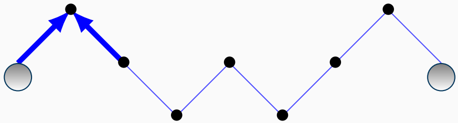

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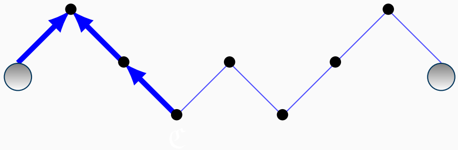

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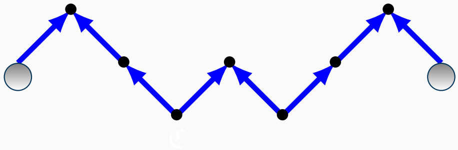

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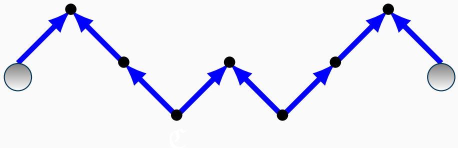

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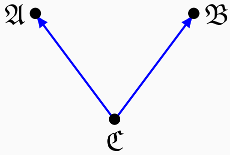

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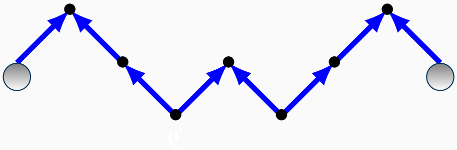

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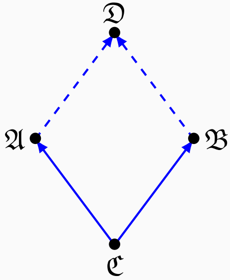

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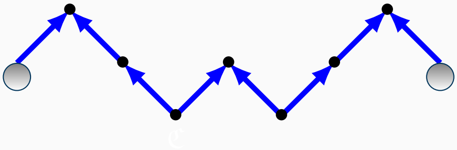

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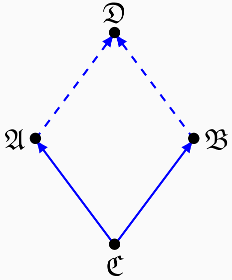

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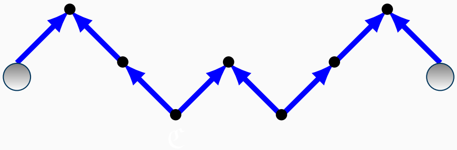

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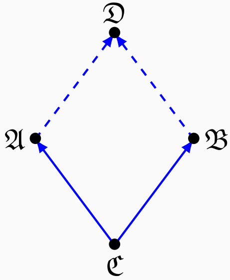

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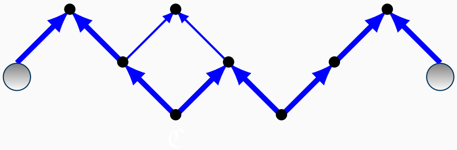

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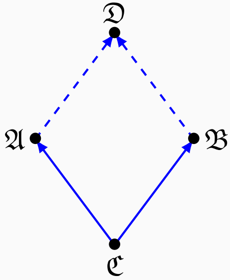

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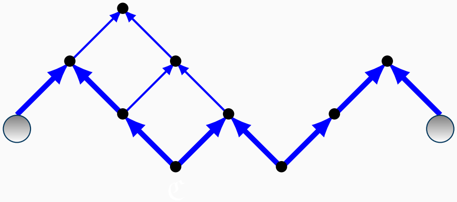

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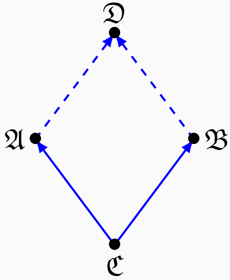

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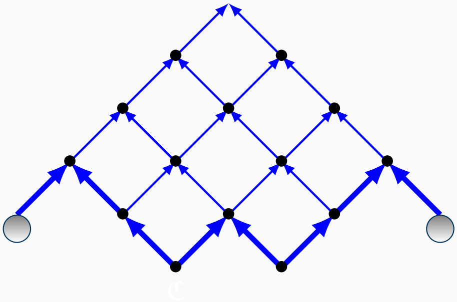

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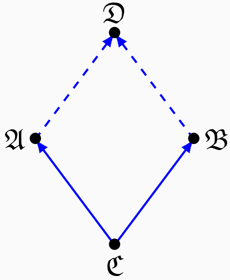

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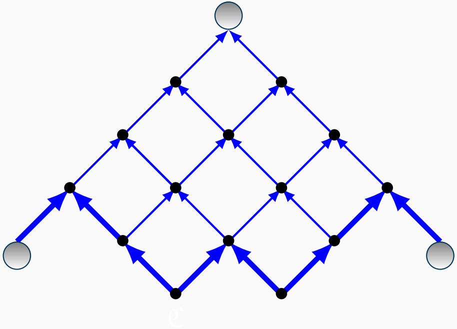

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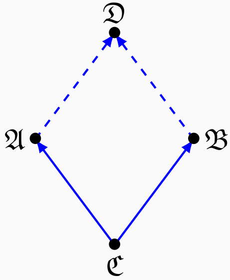

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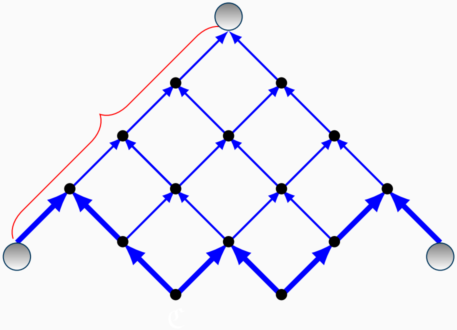

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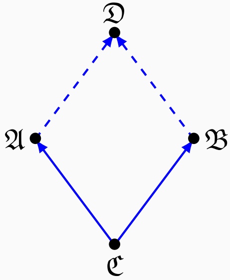

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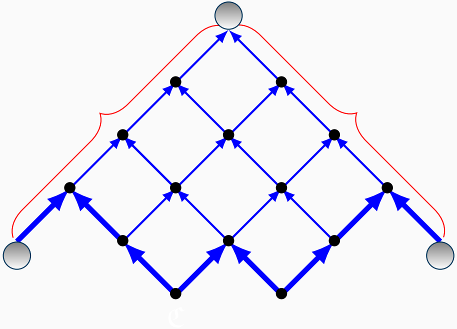

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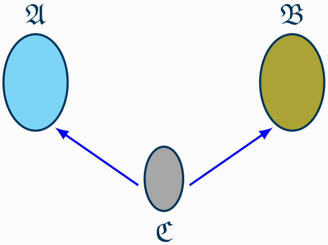

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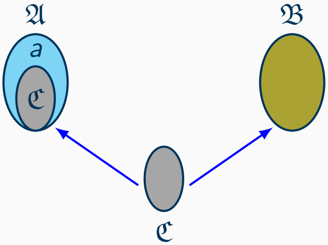

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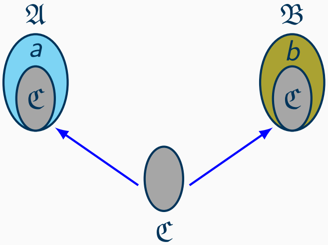

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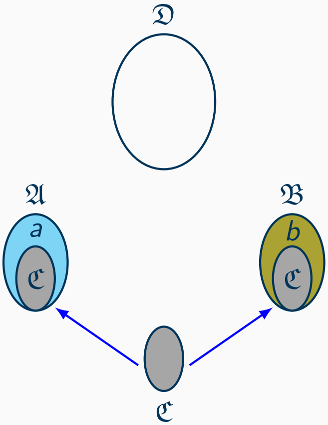

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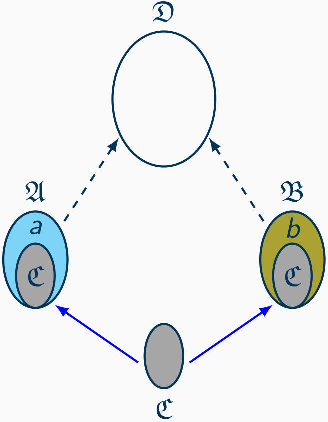

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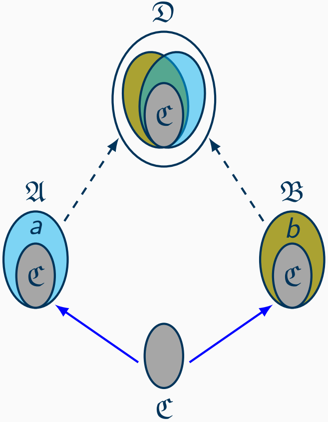

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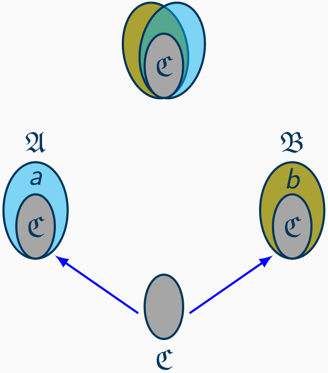

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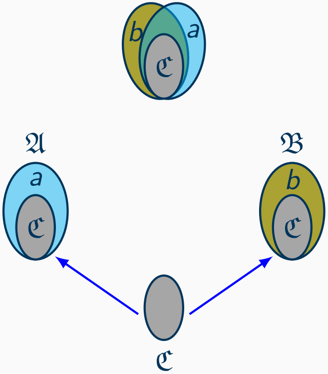

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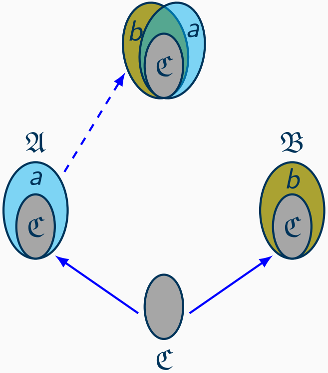

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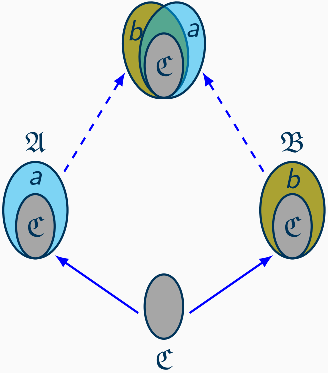

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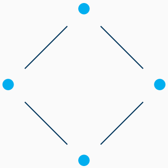

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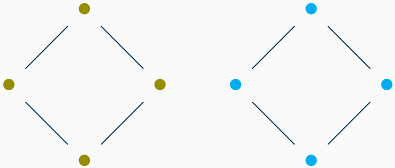

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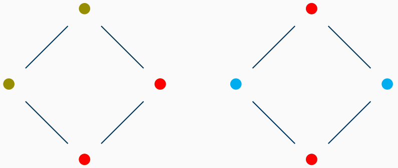

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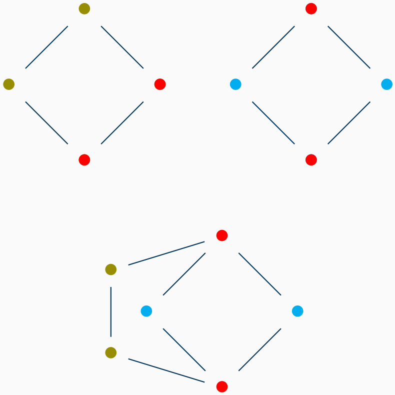

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Example: Subgroups of $\mathbb{Q}/\mathbb{Z}$

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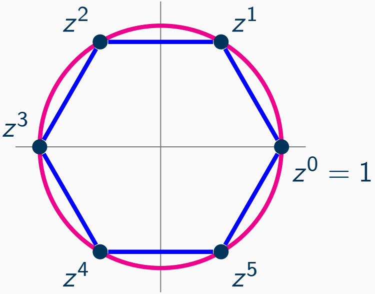

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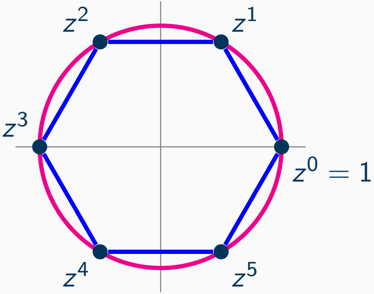

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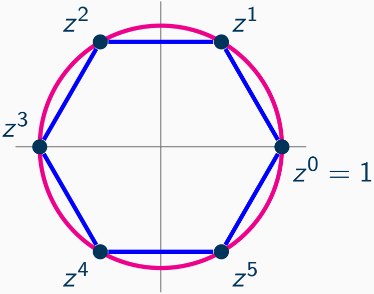

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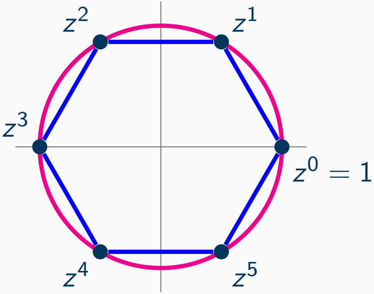

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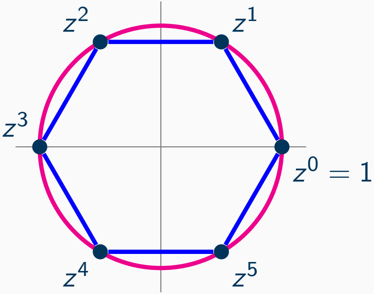

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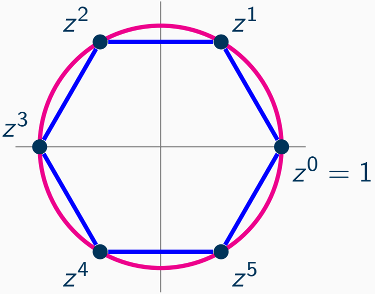

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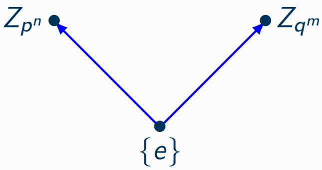

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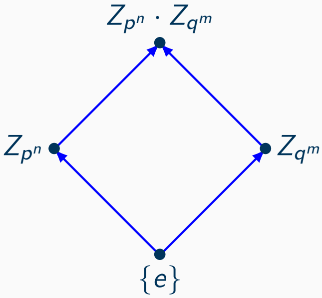

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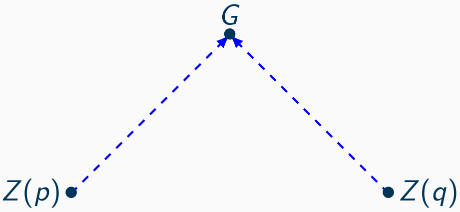

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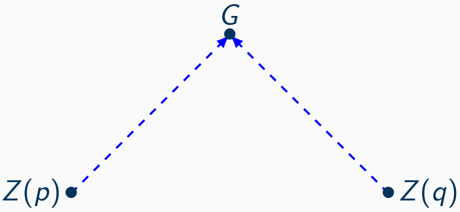

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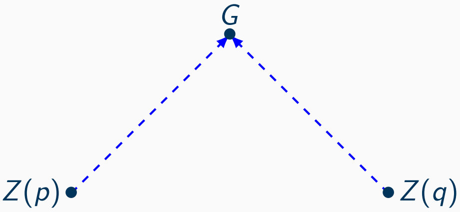

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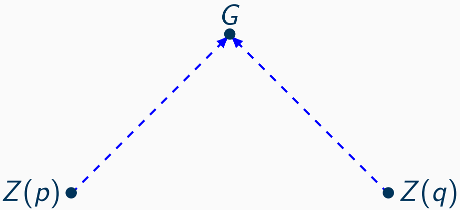

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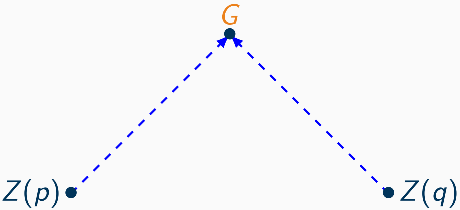

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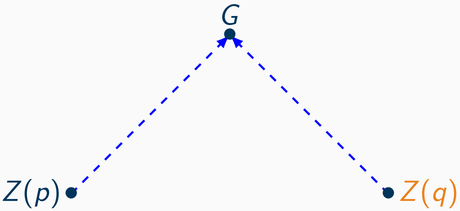

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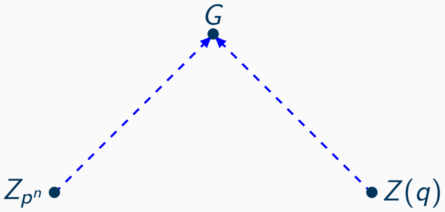

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Monounary Algebras ...

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Definition

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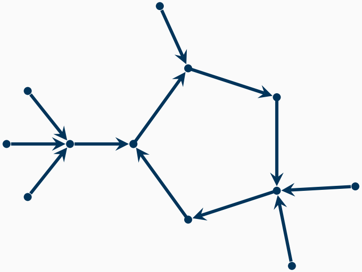

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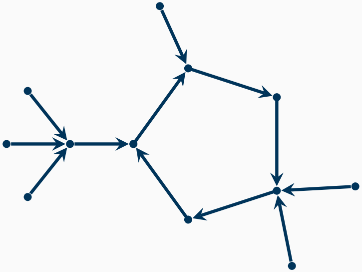

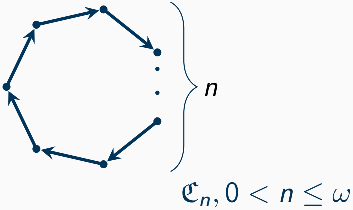

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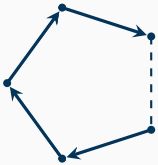

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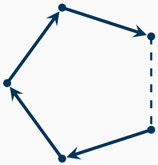

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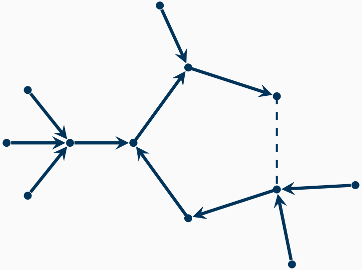

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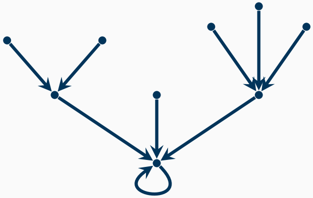

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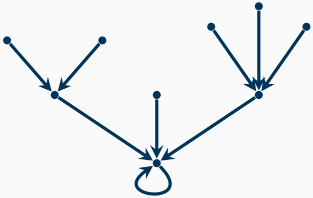

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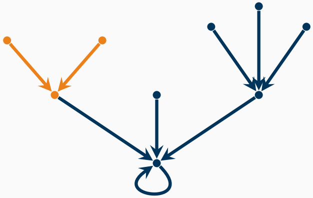

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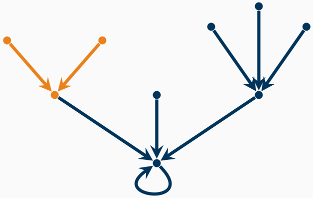

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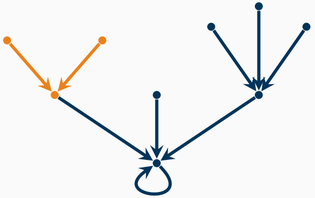

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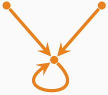

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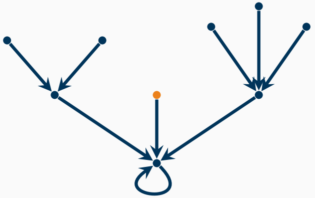

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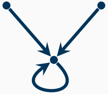

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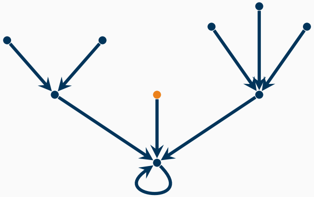

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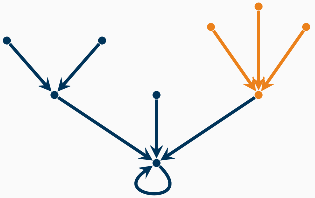

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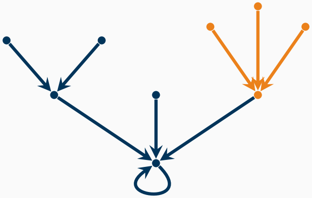

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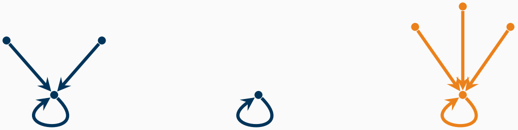

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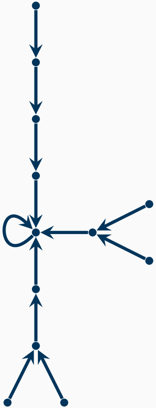
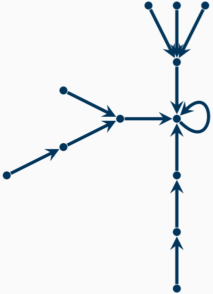
Isomorphic cores: trees



$\mathfrak{A}$



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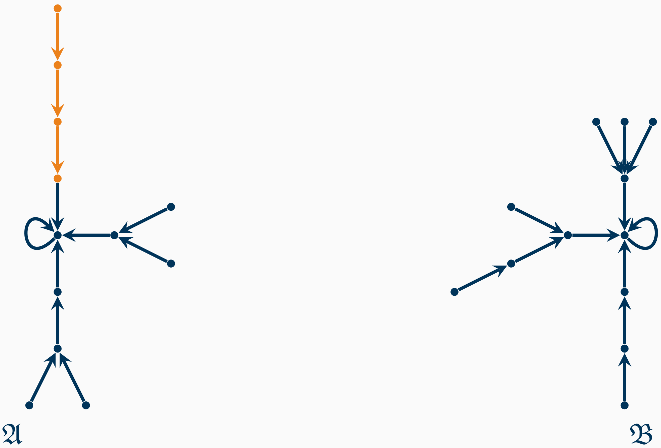

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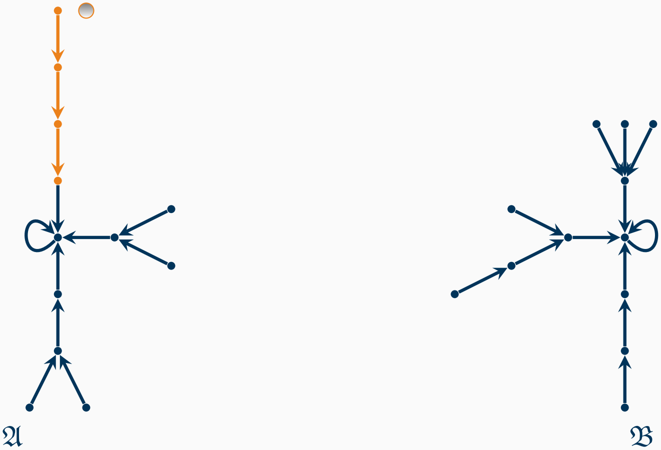

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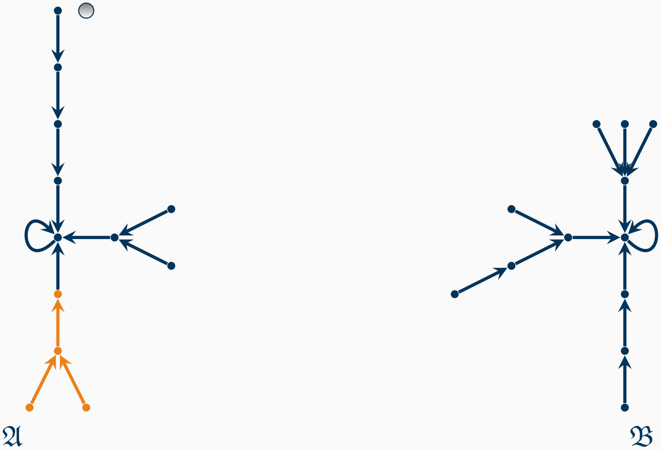

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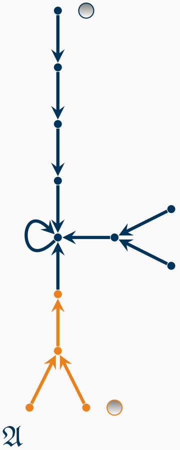

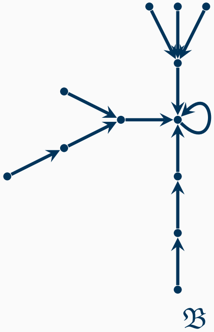

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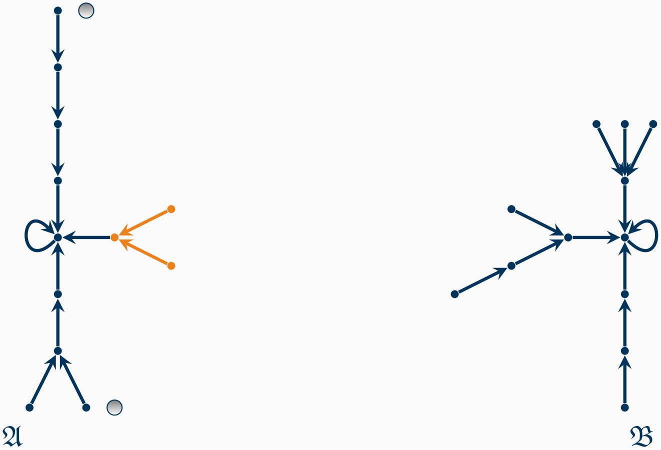

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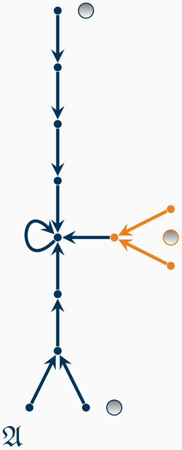

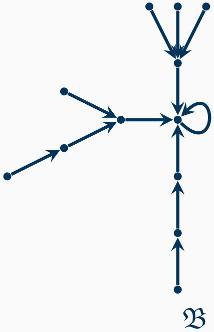

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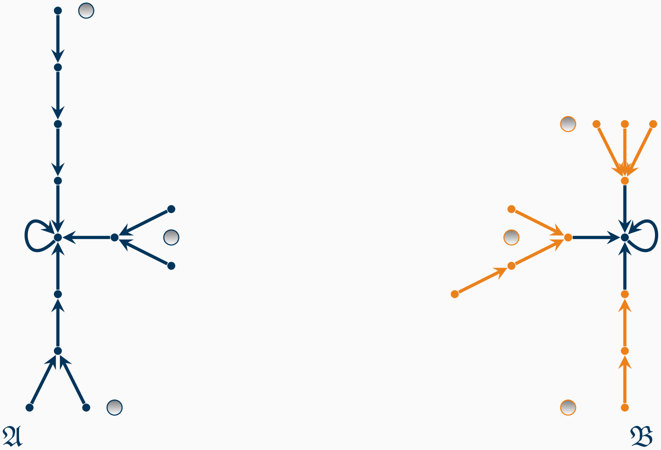

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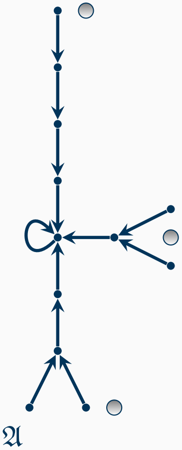

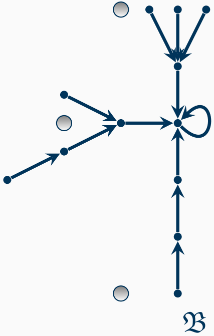

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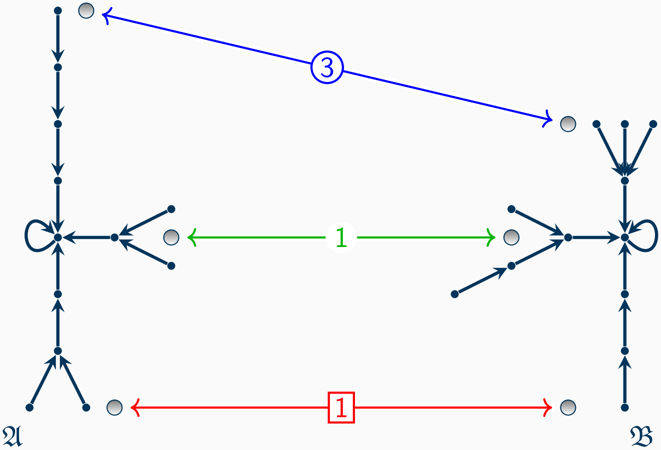

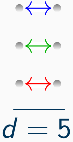

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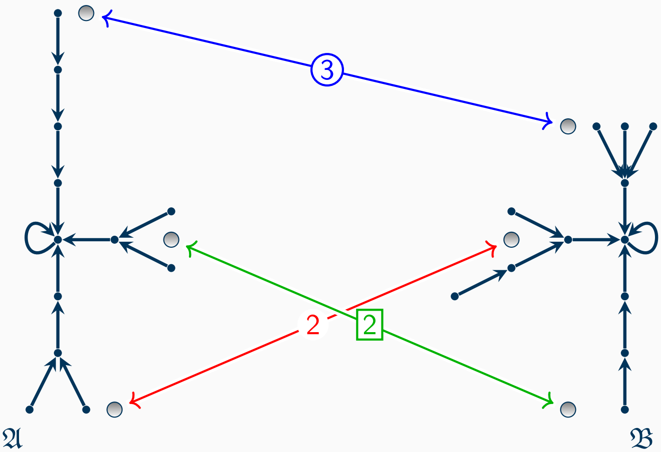

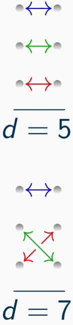

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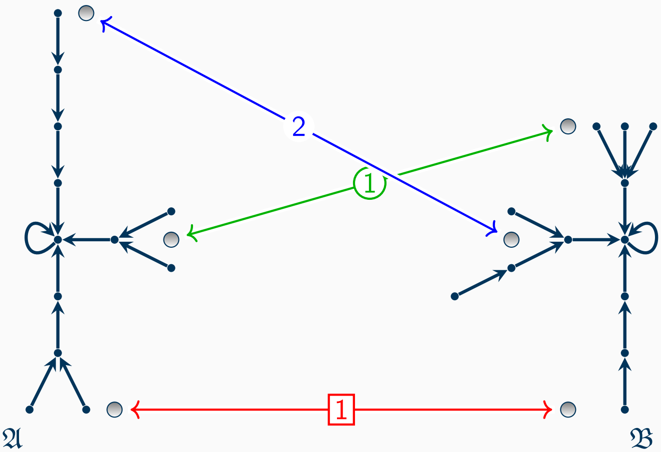

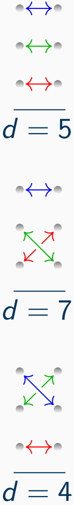

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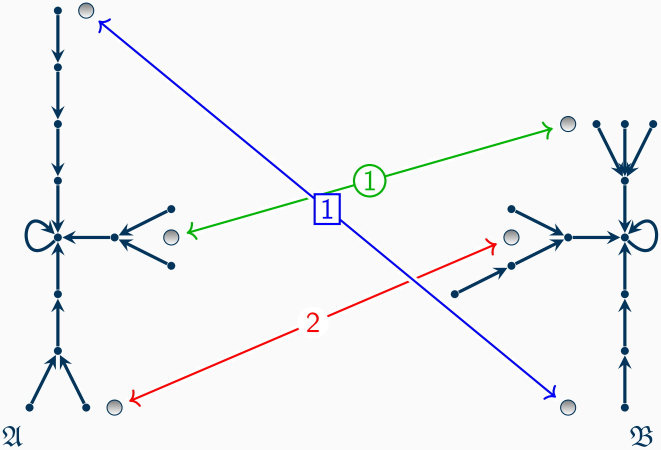

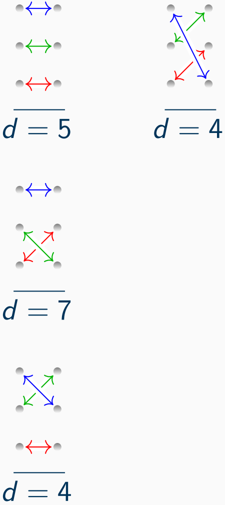

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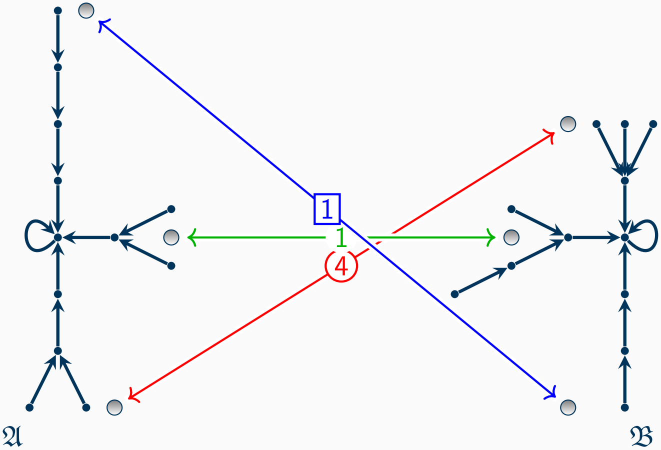

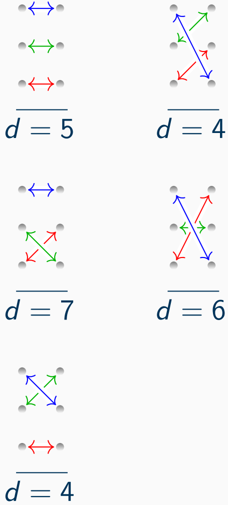

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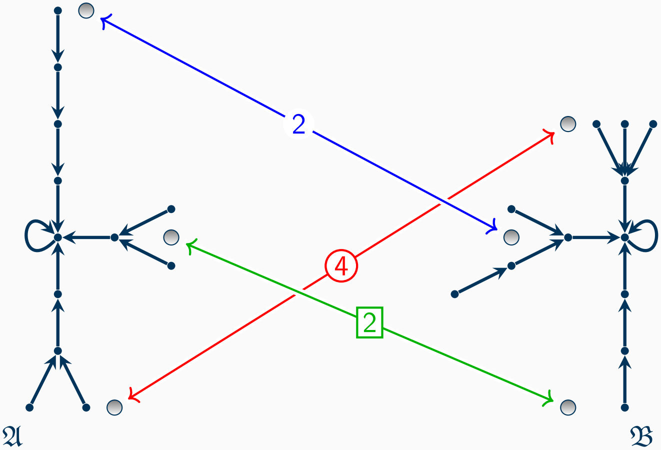

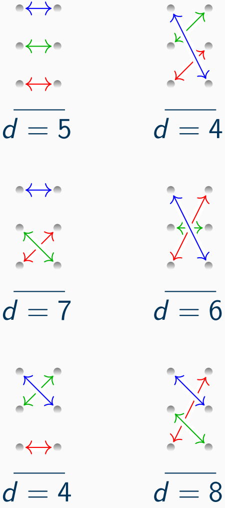

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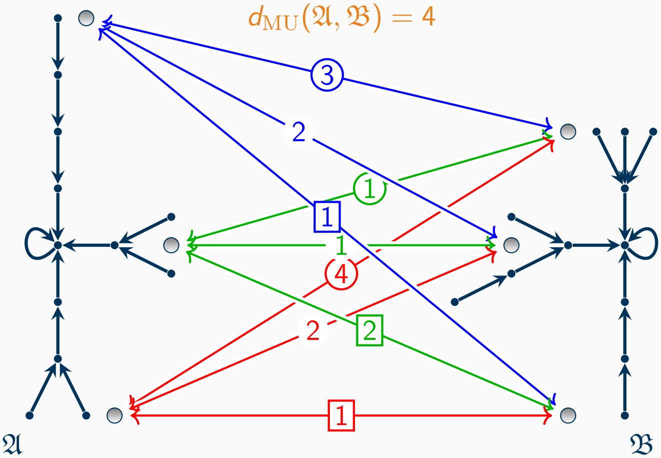
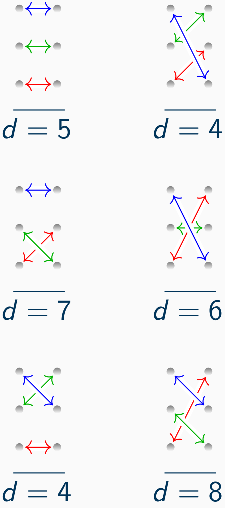
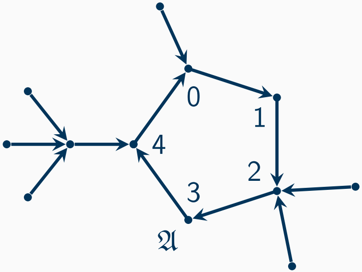




Tree decomposition

Tree decomposition



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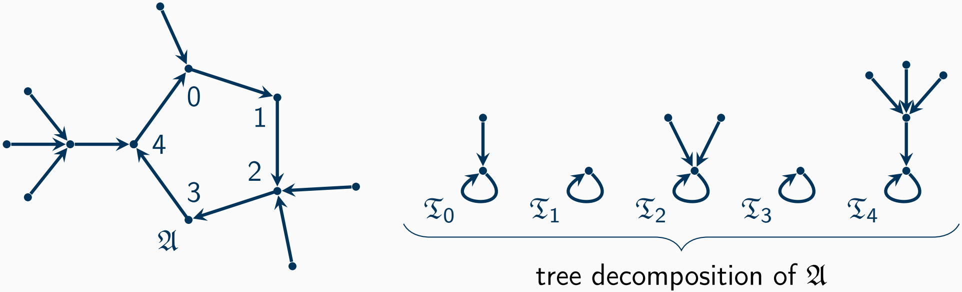

Isomorphic cores: finite cycles

Theorem

Let $\mathfrak{A} = \langle A, f \rangle$ and $\mathfrak{B} = \langle B, g \rangle$ be two connected monounary algebras having cores isomorphic to $\mathfrak{C}_n$, for some finite $n > 0$. Let $\mathfrak{T}_i^A$ and $\mathfrak{T}_i^B$ be their tree decompositions, respectively. Then

$$d_{\mathrm{MU}}(\mathfrak{A}, \mathfrak{B}) = \min_{0 \le k < n} \sum_{\substack{j = i + k \\ \bmod n}} d_{\mathrm{MU}}\left(\mathfrak{T}_i^A, \mathfrak{T}_j^B\right).$$

Open Problems

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- In case of two connected monounary algebras with cores isomorphic to $\mathfrak{C}_\omega$, yet we have no algorithm to calculate the distance.
- Our algorithm does not seem very effective because one has to consider lots of permutations. Is there a more effective algorithm?
- What is the computational complexity of the problem of determining the generator distance between two finite monounary algebras?

Boolean Algebras ...

Boolean algebras

Definition

A Boolean algebra is an algebra of the form

$$\mathfrak{A} = \langle A, +, \cdot, -, 0, 1 \rangle$$

satisfies:

- $+$ and $\cdot$ are commutative and associative, and they distribute over each other.
- $a + 0 = a, \qquad a \cdot 1 = a, \qquad a + \ -a = 1, \qquad a \cdot -a = 0.$
- $a + (b \cdot b) = a, \qquad\qquad a \cdot (a + b) = a.$

$$a \leq b \overset{\text{def}}{\iff} a + b = a \iff a \cdot b = b$$

Atomic Boolean algebras

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An atom in $\mathfrak{A}$ is a minimal non-zero element.

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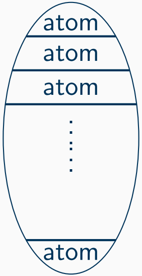

Boolean algebra (the unit)

Finite Boolean Algebras

Finite Boolean Algebras

Lemma

Let $\mathfrak{A}$ and $\mathfrak{B}$ be two finite Boolean algebras with n-many and m-many atoms, respectively. Then,

$$\mathfrak{A} \text{ is largely embeddable into } \mathfrak{B} \iff n \leq m \leq 2n.$$

Finite Boolean Algebras

Lemma

Let $\mathfrak{A}$ and $\mathfrak{B}$ be two finite Boolean algebras with n-many and m-many atoms, respectively. Then,

$$\mathfrak{A} \text{ is largely embeddable into } \mathfrak{B} \iff n \leq m \leq 2n.$$

Theorem

Let $\mathfrak{A}$ and $\mathfrak{B}$ be two finite Boolean algebras with n-many and m-many atoms, respectively. Assume that $n \leq m$, then $d_{\mathrm{BA}}(\mathfrak{A}, \mathfrak{B}) = \lceil \log_2 m - \log_2 n \rceil$.

More on the project

More on the project

- M. Khaled, G. Székely, K. Lefever and M. Friend (2020). DISTANCES BETWEEN FORMAL THEORIES. **The Review of Symbolic Logic**, 13(3), pp. 633 – 654.
- M. Khaled and G. Székely (2021). ALGEBRAS OF CONCEPTS AND THEİR NETWORKS. In: Allahviranloo T., Salahshour S., Arica N. (eds), **Progress in Intelligent Decision Science. IDS 2020. Advances in Intelligent Systems and Computing**, vol 1301. Springer, pp. 611–622.
- T. Aslan, M. Khaled and G. Székely (2021). ON THE NETWORKS OF LARGE EMBEDDINGS. **In preparation**.

Thank you!