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Boolean factors of orthomodular lattices

J. HARDING

For an orthomodular lattice L (abbreviated OML), and $F \subseteq L$, define $C(F) = \{x \in L: x$ commutes with y for all $y \in F\}$. In an unpublished manuscript, G. Bruns, R. Greechie, and L. Herman show that if there exists a finite subset F of L such that $C(F) = C(L)$, then L is isomorphic to the direct product of a Boolean algebra and an OML without non-trivial Boolean factor (see also [1], p. 541). The converse of this statement was also questioned, and is refuted below by counterexample.

As an easy consequence of [2], shown in the above mentioned manuscript, the commutator ideal of an OML L is principal iff L is isomorphic to the direct product of a Boolean algebra and an OML without non-trivial Boolean homomorphic image.

Label MO_2 as in Figure 1, and the four element Boolean algebra as in Figure 2. Define P as the horizontal sum of $(MO_2)^2$ and the four element Boolean algebra, noting that an element of P is either 0, 1, x, x', or an ordered pair of elements of MO_2. Define $L \subseteq P^N$, N the set of all natural numbers, by $z \in L$ iff $\{i \mid z(i) \in \{x, x'\}\}$ is finite. L is a subalgebra of P^N, hence an OML. The commutator ideal of L is principal as 1 is a commutator. $C(L) = \{0, 1\}^N$, but if F is a finite subset of L, there exists $z \in C(F)$ with $z(i) = (0, 1)$ for some $i \in N$, as $(0, 1) \in C((MO_2)^2)$.

L is an OML without non-trivial Boolean homomorphic image, since in every non-trivial homomorphic image of L, 1 is a commutator. However, there exists no finite subset F of L such that $C(F) = C(L)$.

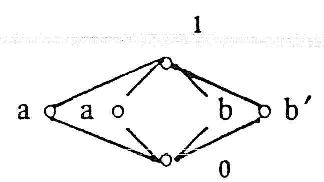

Figure 1

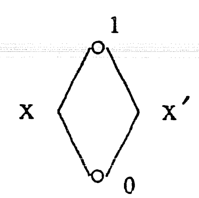

Figure 2

Presented by Alan Day.
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McMaster University
Department of Mathematical Sciences
Hamilton, Ontario